# On-ramp perturbation in the one-lane highway traffic flow model

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2A1170 - Chaos and self-organization

## Introduction

Everyday experiences show that traffic flow has complicated properties that the simple one-line highway model using cellular automata does not reveal. Its most relevant results are certainly the identification of traffic phase transitions : besides freely flowing cars and a full-scale traffic jam, appears what is called synchronized traffic flow. Under certain circumstances, cars would suddenly all slow down to roughly the same speed and tend to stay in lane, indicating that the traffic had jelled into a type of unified, moving mass. This steady state is the result of the self-organized features of the flow. At this precise point, increasing somehow the car density makes a jam, and reducing it turns the flow back to free flow. Hence, it seems that traffic can be in one of three states: free flowing, synchronised or jammed.

However, this self-organized property was established for a simple model and may not support any further constraint, neither any external perturbation to the lane flow. What could be the effects of a on-ramp on a single lane highway on the flow self-organized characteristics ?

## Single-line on-ramp model

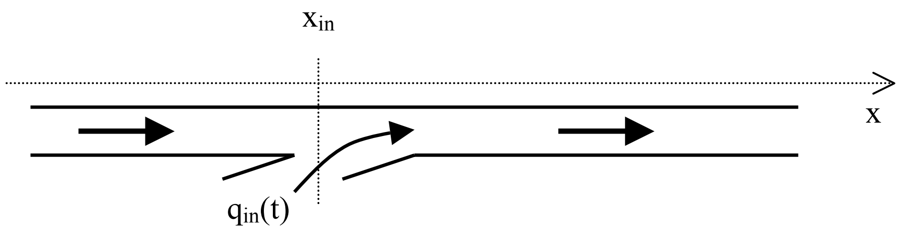


***Fig 1. Schema of a single lane highway with on-and off-ramp.***

The system is a single-lane highway with an on-ramp. For this system, a hydrodynamic model is used, based on Navier-Stockes equation of motion:

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x}\right) = \frac{\rho}{\tau}[V(\rho) - v] - c_0^2\frac{\partial \rho}{\partial x} + \mu\frac{\partial^2 v}{\partial x^2} \qquad (1)$$

where $\rho(x,t)$ is the local vehicle density, $v(x,t)$ is the local velocity, $V(\rho)$ the safety velocity, that is achieved in a time-independent and homogenous traffic flow (a free flow), and $\tau$, $\mu$, $c_o$ specific constants for fluids mechanics. This equation have to be paired with the equation of continuity, which is for this system :

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = q_{\text{in}}(t)\varphi(x - x_{\text{in}}) \qquad (2)$$

where the source term represents the external flux through the on-ramp. Here $\varphi(x)$ describes the spatial distribution of external flux near $x=0$ and normalised so that $q_{in}(t)$ represents the total incoming flux.

## First computation results

The main vehicle flow is a free flow and the model is computed with a constant input flux $q_{in}(t)=f$. For a small $f$, almost no change occurs to the main flow behavior : the homogeneous flow evolves to a slightly modified free flow, where homogeneous regions with different densities are separated by narrow density-rising or descending region near the ramp, the so-called transition layer.

However, for $f$ larger than a critical value $f_c$ a local avalanche-like process occurs at the ramp and a traffic-jam appears. Like every emergent traffic-jam, it is a synchronized flow.

Then, the incoming perturbing flux from the ramp seems to support self-organization. Note that there is no use to simulate the system starting from a congested main flow since self-organisation could never appear from there anyway.

Moreover, another unexpected phenomenon is revealed : for a range of $f$, traffic flow can be either in synchronized self-organized flow or in free flow. This bistability often occurs in solid state physic phase transition when metastable elements are formed. The way to investigate this range of $f$ is to add a perturbation to the constant input flow (just like what is done in solid state physic by adding an impurity perturbing the environment conditions).

## Perturbed input flow

A pulse-type perturbation with a finite amplitude $\delta q_{in}$ for $\delta t$ is applied to the constant flux $f$ which is below the critical input $f_c$ (for $f>f_c$ the stable free flow, where self-organization can appear, does not exist). The result is shown on Fig 2. A localized oscillating state appears from the free flow near the ramp.

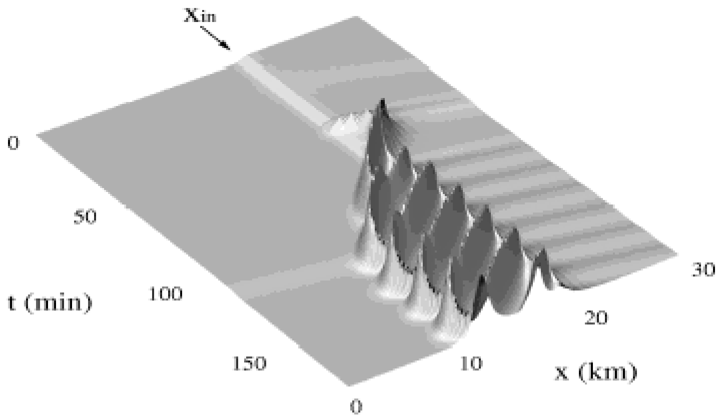


***Fig 2. Birth and evolution of the RH state triggered by a pulse-type perturbation at t=50 min***

After a transient period, the localized oscillation becomes periodic in time (Fig 2). However, this period is independent of the perturbation characteristics ($\delta q_{in}$ and $\delta t$) as long as they are large enough to trigger the transition. Hence, this oscillation is not a transient process but a kind of limit cycle of equations (1) and (2), called “recurring hump” (RH) state. Indeed, this describes a self-excited oscillator where the constant external flux serves as a source for the periodically generated excitations (humps).

Fig 2 shows also that the excitation do not survive far away from the ramp. The main free flow as a density lower than the critical density $\rho_c$ that would turn it into a jam, so the oscillations are absorbed by the main flow that can locally support more cars.

## From free flow to RH state

The transition from the free flow to the RH state needs a closer look-up. If the perturbation is fixed, $\delta q_{in}$ and $\delta t$ are finite constants. Then, for small f, the free flow survives the perturbation (the flow can absorb it). For $f$ larger than a critical value $f_1$, however, the RH state is induced. The backward

transition occurs for a lower critical value of $f$, $f_2<f_1$. The system remains in RH state between $f_2$ and $f_1$, whereas it was still in a free flow configuration in the forward transition (Fig 3).

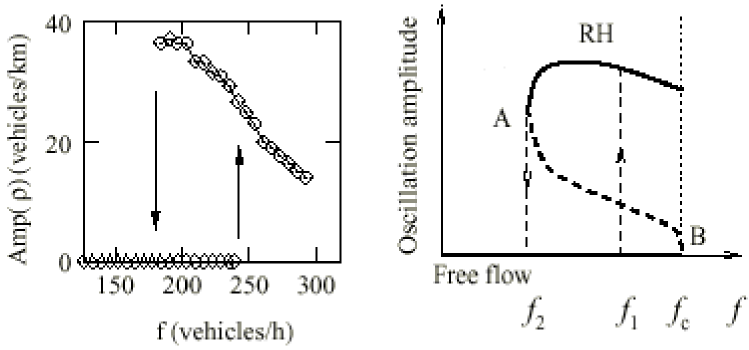


***Fig 3. Transition from free flow to RH state***

This amazing hysteresis transition to RH state is what is actually measured on highways.

It is worth noticing that RH state is really similar to synchronized self-organized flow : discontinuous transition from the free flow to the synchronized flow induced by localized perturbations of finite amplitudes, hysteresis, stability of synchronized flow, gradual spatial transitions from synchronized flow to free flow, and synchronized oscillations. However, where synchronized flow is usually not a stable dynamic phase of traffic flow (because any perturbation turn it either in a free flow or in a jam), RH state are nondecaying near the ramp. RH state is a new state for traffic flow to be added to free flow, synchronized flow and jammed flow.

The system has reached another self-organized state in response of the input perturbation. Self-organization is a feature that can evolve but still remains, whatever external constraint is applied to the system.

## Conclusion

There exists a recurring hump state in highway traffic flow with ramps. In this new state for traffic flow, the density and the flow oscillate periodically and the oscillation are localized near the ramp. This RH state is a stable limit of the nonlinear equations of motion in fluids mechanics (1) and (2). The transition between free flow and RH state shows discontinuity and hysteresis. Many features of the RH state prove that this state is self-organized, but contrary to synchronized flow, it is also stable. Hence, traffic-flow model on a single-line highway with a ramp still reveals self-organization features. Self-organization is a strong characteristic of this system and can not be broken by finite perturbations. Further studies even tend to show that RH state is the origin of synchronized self-organized flow on highways.


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